AB Calculus Review Sheet #5

Review Related Rate Problems.
Review Optimization Problems.
Review Curve Sketching Problems.
Review Separable Differential Equations.
Numerical Methods.

Multiple Choice Questions:

1. A stone is thrown straight up from the top of a building with initial velocity 40 ft/sec and hits the ground 4 seconds later. The height of the building in feet is

   (A) 88 (B) 96 (C) 112 (D) 128 (E) 144

2. The maximum height is reached by the stone (in question 1) after

   (A) $\frac{4}{5}$ sec (B) 4 sec (C) $\frac{5}{4}$ sec (D) $\frac{5}{2}$ sec (E) 2 sec

3. If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, then

   (A) $y^2 = 4\sqrt{x} - 7$ (B) $\ln y = 4\sqrt{x} - 8$ (C) $\ln y = \sqrt{x-2}$

   (D) $y = e^{\sqrt{x}}$ (E) $y = e^{\sqrt{x}-2}$

4. A function $f(x)$ which satisfies the equations $f(x)f'(x) = x$ and $f(0) = 1$ is

   (A) $f(x) = \sqrt{x^2+1}$ (B) $f(x) = \sqrt{1-x^2}$ (C) $f(x) = x$

   (D) $f(x) = e^x$ (E) none of these

5. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 years. After 75 years the ratio of the population P to the initial population P_o is

   (A) $\frac{9}{4}$ (B) $\frac{5}{2}$ (C) $\frac{4}{1}$ (D) $\frac{2\sqrt{2}}{1}$ (E) none of these

6. The Center for Disease Control announced that although there are more AIDS cases reported this year, the rate of increase is slowing down. If we graph the number of AIDS cases as a function of time, the curve is currently

   (A) increasing and linear (B) increasing and concave down

   (C) increasing and concave up (D) decreasing and concave down

   (E) decreasing and concave up

7. The total number of local maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is

   (A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

8. If a particle moves along a horizontal line according to the law $s = t^5 + 5t^4$, then the number of times it reverses direction is

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. Two cars are traveling along perpendicular roads, car A at 40 mi/hr, car B at 60 mi/hr. At noon, when car A reaches the intersection, car B is 90 miles away, and moving toward the intersection. At 1 pm the distance between the cars is changing at the rate of

   (A) –20 mph (B) 6.8 mph (C) 0.4 mph (D) –0.4 mph (E) 2 mph

10. A balloon is being filled with helium at the rate of $4\pi\ \text{ft}^3/\text{min}$. The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}\ \text{ft}^3/\text{min}$ is

    (A) 4π (B) 8π (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$

11. A vertical circular cylinder has radius r feet and height h feet. If the height and radius both increase at the constant rate of 2 ft/sec, then the rate, in square feet per second, at which the lateral surface area increases is

    (A) $4\pi r$ (B) $2\pi(r+h)$ (C) $4\pi(r+h)$ (D) $4\pi rh$ (E) $4\pi h$

12. A local minimum value of the function $y = \frac{e^x}{x}$ is

    (A) $\frac{1}{e}$ (B) 1 (C) -1 (D) e (E) 0

13. The area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve $y = e^{-x^2}$ is

(A) $\sqrt{\frac{2}{e}}$ (B) $\sqrt{2e}$ (C) $\frac{2}{e}$ (D) $\frac{1}{\sqrt{2e}}$ (E) $\frac{2}{e^2}$

14. The table below shows values of $f''(x)$ for various values of x:

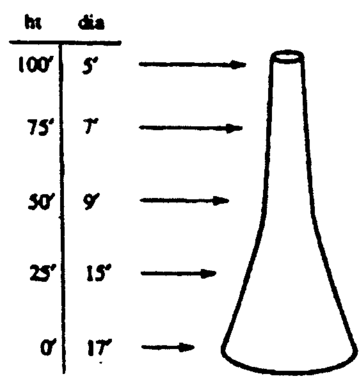
x	-1	0	1	2	3
$f''(x)$	-4	-1	2	5	8

The function could be

(A) a linear function (B) a quadratic function (C) a cubic function

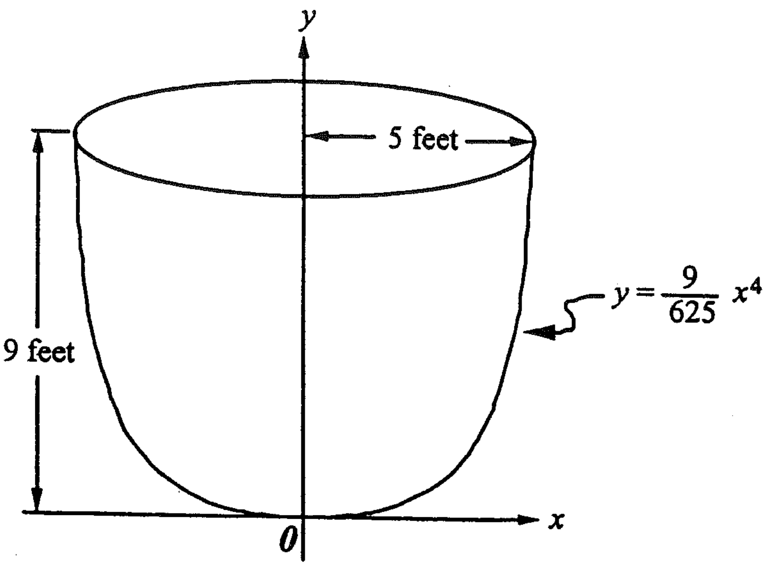
(D) a fourth-degree function (E) an exponential function

15. A smokestack 100 ft tall is used to treat industrial emissions. The diameters, measured at 25-foot intervals, are shown in the drawing. Using the Midpoint rule, estimate the volume of the smokestack to the nearest 100 ft^3.



(A) 8100 ft^3 (B) 9500 ft^3 (C) 9800 ft^3 (D) 12,500 ft^3 (E) 39,300 ft^3

Free Response Questions:



1. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate if 8 cubic feet per minute.

   (a) Find the volume of the tank. Indicate units of measure.

   (b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?

   (c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.

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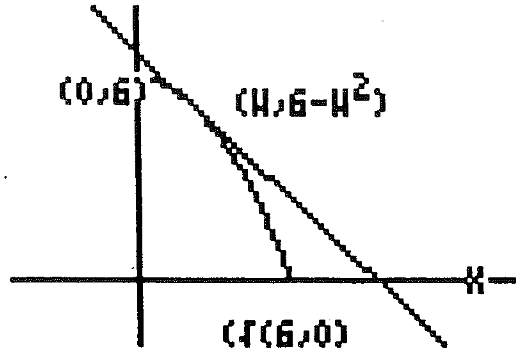
2. Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.

   (a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

   (b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

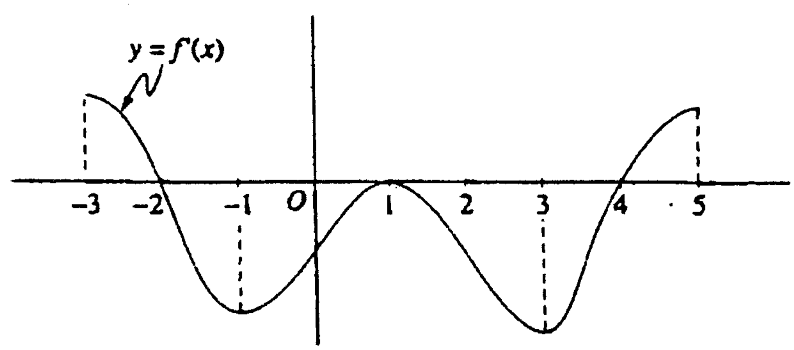
3.



Let $f(x) = 6 - x^2$, For $0 < w < \sqrt{6}$, let $A(w)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(w, 6 - w^2)$. (See figure above.)

(a) Find $A(1)$.

(b) For what value of w is $A(w)$ a minimum?



4. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

   (a) For what values of x does f have a relative maximum? Why?

   (b) For what values of x does f have a relative minimum? Why?

   (c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

   (d) Suppose that $f(1) = 0$. Draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

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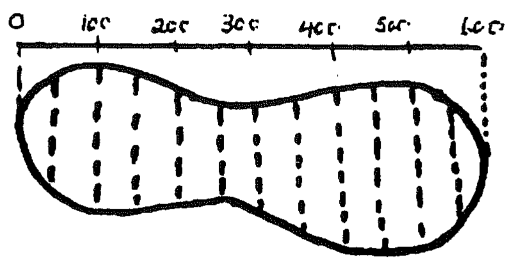
5. The figure drawn below shows a tract of land with measurements in feet. A surveyor has measured it width at 50-ft intervals with the results shown in the table. Use the trapezoidal rule to approximate th acreage of the field. Note: 43560 square feet = 1 acre.



x	0	50	100	150	200	250	300	350	400	450	500	550	600
w	0	165	192	146	63	42	84	155	224	270	267	215	0

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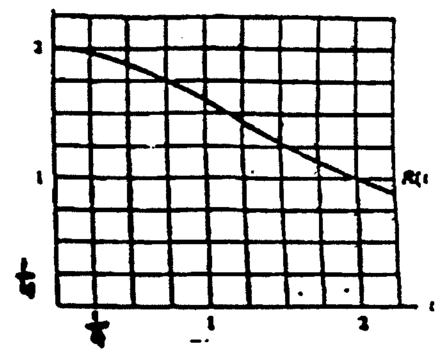
6. Water is leaking out of a tank at a rate of $R(t)$ gallons/hour, where t is measured in hours.



a. Write a definite integral that expresses the total amount of water that leaks out in the first two hours.

b. The figure given at the right is the graph of $R(t)$. On the sketch, shade in the region which represents the total amount that leaks out I the first two hours.

c. Give an upper estimate and a lower estimate of the total amount that leaks out in the first two hours.

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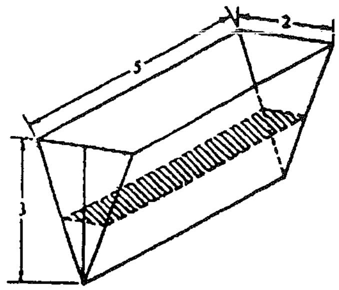
7. The amount of money required to finance the digging of a tunnel equals the length of the tunnel times the cost per unit length. If the cost is constant, calculating the cost to dig a tunnel poses no problem; however, the price per unit length increases as the tunnel gets longer because of the expense of carrying in workers and tools and carrying out dirt and rock. Assume that the cost per foot varies according to the table below.

Length	0 - 100	100 - 200	200 - 300	300 - 400	400 - 500
Cost/ft	\$500	\$820	\$1180	\$1480	\$2020

Length	500 - 600	600 - 700	700 - 800	800 - 900	900 - 1000
Cost/ft	\$2500	\$3020	\$3580	\$4180	\$4820

a. Find the total cost for digging a tunnel 1000 feet long.

b. How much money can be saved by starting the 1000-foot tunnel from both ends and making halves meet in the middle?

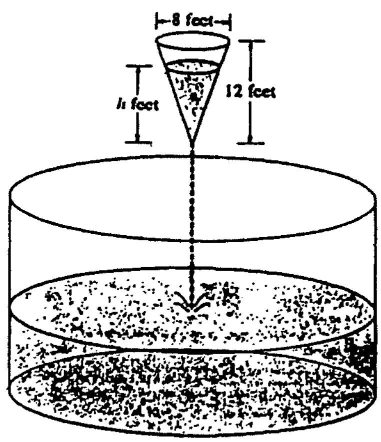


8. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

   (a) Find the volume of water in the trough when it is full.

   (b) What is the rate of change in h when the trough is $\frac{1}{4}$ full by volume?

   (c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?



9. As shown I the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V=\frac{1}{3}\pi r^2 h$.)

   (a) Write an expression for the volume of water in the conical tank as a function of h.

   (b) At what rate is the volume of water in the conical tank changing when $h=3$? Indicate units of measure.

   (c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h=3$? Indicate the units of measure.

10. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variable to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity in defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

Review Sheet #5

1. B	6. B	11. C
2. C	7. B	12. D
3. E	8. C	13. A
4. A	9. D	14. C
5. D	10. A	15. C

1. 1996 AB5
2. 1993 AB4
3. 1994 BC4
4. 1996 AB1
5. See attached
6. See attached
7. See attached
8. 1987 AB5
9. 1995 AB5
10. 1997 AB6

1 1996 AB5

Solution

(a) volume $= V = \pi \int_0^9 \frac{25}{3}\sqrt{y}\,dy = 150\pi$ ft^3

(or 471.238 ft^3 or 471.239 ft^3)

$$V = 2\pi \int_0^5 x\left(9 - \frac{9}{625}x^4\right) dx$$

(b) time $= \frac{\text{volume}}{\text{rate}} = \frac{150\pi}{8}$

therefore, 59 minutes

(c) $V = \pi \int_0^h \frac{25}{3}\sqrt{y}\,dy$

$$\frac{dV}{dt} = \frac{25}{3}\pi\sqrt{h}\,\frac{dh}{dt}$$

$$\frac{dV}{dt} = 8$$

when $h = 4$, $8 = \frac{25}{3}\pi(2)\frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{12}{25\pi} \text{ ft/min}$$

(or 0.152 ft/min or 0.153 ft/min)

Scoring Scale

Points

(a) 3 $\begin{cases} 2: \text{integral} \begin{cases} 1: \text{limits and } \pi/2\pi \\ \quad \pi\int_0^9 (\ )\,dy, \quad 2\pi\int_0^5 (\ )\,dx \\ 1: \text{integrand} \end{cases} \\ 1: \text{answer with units} \end{cases}$

not eligible for answer point unless first two points are earned.

(b) 1: integer answer

(c) 5 $\begin{cases} 1: \text{volume as definite integral using } h \\ 2: \text{finding } \frac{dV}{dt} \begin{cases} 1: \frac{dV}{dh} \\ 1: \frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} \end{cases} \\ 1: \frac{dV}{dt} = 8 \\ 1: \text{answer with units} \end{cases}$

if V is linear, max 2/5 (0 - 0 - 1 - 1 - 0)
if V is constant, max 1/5 (0 - 0 - 0 - 1 - 0)

Question 4

This problem, also common to both exams, asks some familiar questions about a not-so-familiar function. In their solution, students must demonstrate some facility with both logarithmic and trigonometric functions, particularly the latter.

The question is phrased in such a manner that the minimum value of the function occurs at an interior point of the interval and the maximum value of the function occurs at the end points of the interval. Many students failed to recognize that the domain was specified as $[\pi, 2\pi]$ and consequently lost all the points dependent on end-point analysis. Furthermore, many students were unable to distinguish among a value of the function, a number in the domain of the function, and a point on the graph; their confusion often cost them the answer point in Part (a).

4. Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.

   (a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

   (b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

Solution | *Scoring Scale*

Points

(a) $f'(x) = \left(\frac{1}{2+\sin x}\right)\cos x$;

In $[\pi, 2\pi]$, $\cos x = 0$ when $x = \frac{3\pi}{2}$;

x	$f(x)$
π	$\ln(2) \doteq 0.693$
2π	$\ln(2)$
$\frac{3\pi}{2}$	$\ln(1) = 0$

Absolute maximum value is ln 2.

Absolute minimum value is 0.

5 points:

1: Finds $f'(x)$

1: Solutions for $f'(x) = 0$, including at least one solution in $[\pi, 2\pi]$

1: Evaluates f at π and 2π

1: Evaluates f at student's critical point(s) in $[\pi, 2\pi]$

< −1 > Not explicitly excluding all evaluations at x outside $[\pi, 2\pi]$ and all values of $f(x)$ outside $[0, \ln 2]$

1: Answers; must come from evaluations at endpoints and student's critical point(s) in $[\pi, 2\pi]$

< −1 > If x-value or ordered pair

NOTE: If student uses degree measures of critical points or end points and does not give radian equivalents then student loses 1 point the first time this occurs.

(b) $$f''(x) = \frac{(-\sin x)(2+\sin x) - \cos x \cos x}{(2+\sin x)^2}$$
$$= \frac{-2\sin x - 1}{(2+\sin x)^2};$$

$f''(x) = 0$ when $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

	π		$\frac{7\pi}{6}$		$\frac{11\pi}{6}$		2π
Sign of f''		−		+		−	
Concavity		down		up		down	

$x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ since concavity changes as indicated at these points.

4 points:

1: Finds $f''(x)$

2: Solutions for $f''(x) = 0$
1 point for each solution in $[\pi, 2\pi]$
< −1 > For each incorrect solution
< −1 > For solutions outside $[\pi, 2\pi]$

NOTE: 1 of 2 if student's $f''(x) \neq 0$ in $[\pi, 2\pi]$ and student states that

1: Specific justification

1994 BC 4

3

Solution | Points | *Scoring Scale*

(a)

$$f(x)=6-x^2;\ f'(x)=-2x$$
$$f'(1)=-2$$
$$y-5=-2(x-1) \text{ or } y=-2x+7$$
$$x\text{-int: } \frac{7}{2} \qquad y\text{-int: } 7$$
$$A(1)=\frac{1}{2}\left(\frac{7}{2}\right)(7)=\frac{49}{4}$$

3 points:
- 1: $f'(1)=-2$
- 1: Finds equation of line or x-intercept or y-intercept
- 1: Answer

(b)

$$f'(w)=-2w;\quad y-(6-w^2)=-2w(x-w)$$
$$x\text{-int: } \frac{6+w^2}{2w} \qquad y\text{-int: } 6+w^2$$
$$A(w)=\frac{(6+w^2)^2}{4w}$$
$$A'(w)=\frac{4w\big(2(6+w^2)(2w)\big)-4(6+w^2)^2}{16w^2}$$
$$A'(w)=0 \text{ when } (6+w^2)(3w^2-6)=0$$
$$w=\sqrt{2}$$

A': $-$ on $(0, \sqrt{2})$, $+$ on $(\sqrt{2}, \sqrt{6})$; axis w marked at 0, $\sqrt{2}$, $\sqrt{6}$

6 points:
- 1: Equation of line
- 1: Expresses x-intercept or y-intercept in terms of w
- 1: $A(w)$
- 1: $A'(w)$
- 1: Solves $A'(w)=0$
- 1: Shows solution yields a minimum

Note: $A(w)$ must be of the form $\dfrac{P(w)}{Q(w)}$, neither constant, to be eligible for derivative point.

1996 AB

(4)

Solution

Scoring Scale

Points

(a) $x = -2$

$f'(x)$ changes from positive to negative at $x = -2$, or

f is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.

$$2 \begin{cases} 1 : x = -2 \\ 1 : \text{reason} \end{cases}$$

(b) $x = 4$

$f'(x)$ changes from negative to positive at $x = 4$, or

f is decreasing to the left of $x = 4$ and increasing to the right of $x = 4$.

$$2 \begin{cases} 1 : x = 4 \\ 1 : \text{reason} \end{cases}$$

(c) $(-1, 1)$ and $(3, 5)$

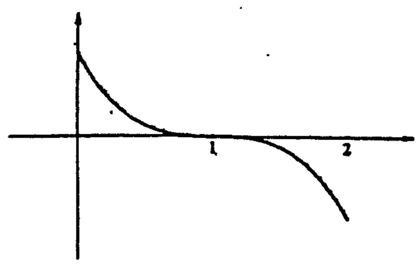
f' is increasing on these intervals.

$$3 \begin{cases} 1 : (-1, 1) \\ 1 : (3, 5) \\ 1 : \text{reason using } f' \end{cases}$$

max 1/3 for one incorrect interval

0/3 for two incorrect intervals

(d)



$$2 \begin{cases} <-1> \ f(1) \neq 0 \\ <-1> \ \text{not decreasing} \\ <-1> \ \text{incorrect concavity} \\ <-1> \ f'(1) \neq 0 \end{cases}$$

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Review Sheet # 5 Answers

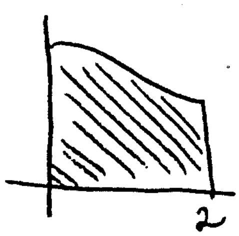
(5) $\frac{1}{2}(50)\left[0+2(165)+2(192)+2(146)+2(63)+2(42)+2(84)+2(155)+2(224)+2(270)+2(267)+2(215)+0\right]=$

$= 88300$ sq. feet

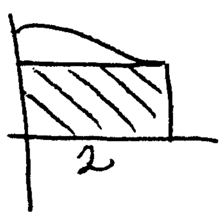
$= 2.027$ acres

(6) (a) $\int_0^2 R(t)\,dt$

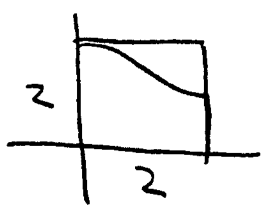
(b)



(c) 1.

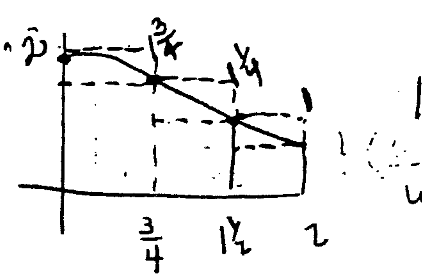


2 gal



4 gal. rough estimate

Between 2 and 4 gallon leak out of the tank in the 2 hours.



lower sums $\frac{3}{4}\left(1\frac{3}{4}\right)+\frac{3}{4}\left(1\frac{1}{4}\right)+\frac{1}{2}(1)=2.7375$

upper sums $\frac{3}{4}(2)+\frac{3}{4}\left(1\frac{3}{4}\right)+\frac{1}{2}\left(1\frac{1}{4}\right)=3.4375$

Between 2.7375 and 3.4375 gallons will leak out of the tank in the 2 hours.

Review #5 question 7

(7) Cost = $100\,[500 + 820 + 1180 + 1480 + 2020 + 2500 + 3020 + 3580 + 4180 + 4820] = 100\,(24100)$

$= \$2{,}410{,}000$

The cost will be \$2,410,000. Note the trapezoidal rule does not apply because the price changes at each 100 ft section.

(b) $2 \cdot 100\,[500 + 820 + 1180 + 1480 + 2020] = \$1{,}200{,}000.$

Note this is a very large savings!!

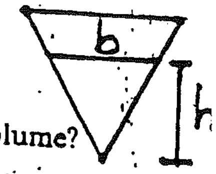
AB-5
1987

The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted ... nsc triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the r ... of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

(b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?



a) $\frac{1}{2}\cdot 2\cdot 3\cdot 5 = 15$ 1 { Answer

b) $V = \frac{5}{2} bh$

$\left[\frac{b}{h} = \frac{2}{3}\right]$

$V = \frac{5}{3} h^2$

$\frac{dV}{dt} = \frac{10}{3} h \frac{dh}{dt}$

$\left[\begin{array}{l} \frac{15}{4} = \frac{5}{3} h^2 \\ h = \frac{3}{2} \end{array}\right]$

$-2 = \frac{10}{3} \cdot \frac{3}{2} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-2}{5}$

5 {
1: Correct V in terms of 1 or 2 variables, wherever it appears ($\frac{1}{2} bh\ell$: 0/1)
1: $\frac{b}{h} = \frac{2}{3}$, or equivalent
1: Correct differentiation with respect to time
1: $h = \frac{3}{2}$ (or $b = 1$, in context)
1: $\frac{dh}{dt} = \frac{-2}{5}$ ($\frac{dh}{dt} = \frac{2}{5}$ also acceptable.)

c) $A = 5b = \frac{10}{3} h$

$\frac{dA}{dt} = \frac{10}{3} \frac{dh}{dt}$

$\frac{dA}{dt} = \frac{10}{3} \cdot \frac{-2}{5} = \frac{-4}{3}$

3 {
1: A as a function of 1 or 2 variables, explicit or implicit
1: $\frac{dA}{dt}$
1: Answer

<u>NOTE</u>: If student sets a variable equal to a constant before differentiating:

In (b), max 3/5: { 1 formula; 1 $\frac{b}{h} = \frac{2}{3}$ } In (c), max 1/3: { 1 or

9

Solution

(a) $\frac{r}{h} = \frac{4}{12} = \frac{1}{3} \qquad r = \frac{1}{3}h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{\pi h^3}{27}$

(b) $\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt} = \frac{\pi h^2}{9}(h-12) = -9\pi$

V is decreasing at 9π ft^3/min

(c) Let W = volume of water in cylindrical tank

$W = 400\pi y; \ \frac{dW}{dt} = 400\pi\frac{dy}{dt}$

$400\pi\frac{dy}{dt} = 9\pi$

y is increasing at $\frac{9}{400}$ ft/min

Scoring Scale

Points

2 $\begin{cases} 1\text{: } r = \frac{1}{3}h \\ 1\text{: } V \text{ as a function of } h \\ \underline{\text{Note}}\text{: 0/2 if } r \text{ constant} \end{cases}$

3 $\begin{cases} 1\text{: } \frac{dV}{dt} \text{ using chain rule} \\ 1\text{: } \frac{dh}{dt} = h - 12 \\ 1\text{: Solves for } \frac{dV}{dt} \text{ and gives answer with units} \\ \underline{\text{Note}}\text{: 0/1 if } \frac{dV}{dt} > 0 \end{cases}$

4 $\begin{cases} 1\text{: } W \text{ as a function of } y \\ 1\text{: } \frac{dW}{dt} = 400\pi\frac{dy}{dt} \\ 1\text{: } \frac{dW}{dt} = \left|\frac{dV}{dt}\right| \text{ or } -\frac{dV}{dt} \\ 1\text{: Solves for } \frac{dy}{dt} \text{ and gives answer with units} \end{cases}$

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~~6.~~ Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

(a) $\dfrac{dv}{dt} = -2v - 32 = -2(v+16)$

$$\frac{dv}{v+16} = -2\,dt$$

$$\ln|v+16| = -2t + A$$

$$|v+16| = e^{-2t+A} = e^{A}e^{-2t}$$

$$v + 16 = Ce^{-2t}$$

$$-50 + 16 = Ce^{0}\,; \quad C = -34$$

$$v = -34e^{-2t} - 16$$

6 $\begin{cases} 1\text{: separates variables} \\ 1\text{: antiderivative of } dv \text{ side} \\ \quad 0/1 \text{ if not } \int \frac{dv}{av+b},\ a \neq 0 \\ 1\text{: antiderivative of } dt \text{ side} \\ 1\text{: constant of integration} \\ 1\text{: uses initial condition } v(0) = -50 \\ 1\text{: solves for } v(t) \\ \quad 0/1 \text{ if not solving } \frac{dv}{av+b} = k\,dt \\ \quad \text{where } a,\ b,\ k \text{ nonzero} \\ \quad 0/1 \text{ if no constant of integration} \end{cases}$

0/6 if variables not separated

(b) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(-34e^{-2t} - 16\right) = -16$

1: limit value
must be exponential $v(t)$ with finite limit

(c) $v(t) = -34e^{-2t} - 16 = -20$

$$e^{-2t} = \frac{2}{17}\,; \quad t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$$

2 $\begin{cases} 1\text{: sets } v(t) = -20 \\ 1\text{: solution} \\ \quad \text{must be exponential } v(t) \end{cases}$

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